Comparing companies using linear equations

Music Inc. and RadioNow are two companies that carry a new style of mp3 player. During each company's annual executive meeting, they re-evaluated their spending as well as their market prices and profits. Currently, Music Inc. expenses can be modeled by the function $E(x)=500+20x$ where x is the numbers of mp3 players produced. Their profit from each mp3 player is modeled by the function $P(x)=45x$ where x is the number of mp3 players sold at $45 each. The expenses of RadioNow can be modeled by the function $C(x)=500+15x$ where x is the number of mp3 players sold. The profit from each mp3 player made by RadioNow is modeled by the function $M(x)=50x$ where x is the number of mp3 players sold at $50 each.

I. Graph the four functions *E(x), P(x), C(x) and M(x)* on your graphing calculator. Use the following window:

y-

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WINDOW
Xmin=-10
Xmax=40
Xscl=1
Ymin=-50
Ymax=1000
Yscl=1
Xres=1
```

Sketch each equation on the graph below, paying careful attention to the intercept and the points of intersection between all of the lines.
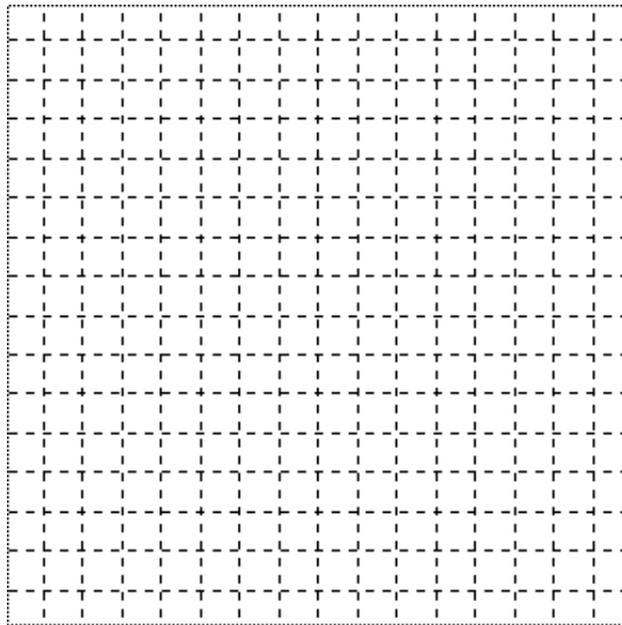
II. Using your graph and graphing calculator, and solving using inverse relationships, answer each of the following questions.

1. What does it mean to *break even*? Which company reaches this point first and how many mp3 players were sold?

2. Where does each company break even? What does this look like on the graph? Write the equation used to solve this and then show your work.

3. Over time who makes the most profit? How is this seen on the graph? Equations?

4. Where do the expense equations intersect? What does this mean and how is this reflected in the equation?

5. Where do the profit equations intersect? How is this reflected in the equations?

6. What is the profit if each company sells 100 mp3 players? 200 mp3 players? You may use your calculator to solve the problem but also show the setup and work for solving by hand.
   100 mp3 players: Music Inc. _____   RadioNow _____

   200 mp3 players: Music Inc. _____   RadioNow _____

7. A third company is also selling the new mp3 player for $60. What would the profit equation look like? How will this company compare to the other two? Profit? How is this seen on the graph?

8. If Music Inc. and RadioNow made $2000, how many mp3 players did they sell? (Ignore expenses. Use only profit equations.)

9. After selling 10 mp3 players, what is each company's profits (ignoring expenses again)? Is this before or after breaking even?

10. When does each company make more than $300? Write an inequality to model this situation.

11. When does each company make less than $750? Write an inequality to model this situation.

12. Why is it important to investigate profit vs. expense and also other trends?

13. As a company what do you want to happen to expenses? Profit?

14. Which company would you choose to work for and why? What factors affect a company's start-up costs?

Comparing companies using linear equations (answer key)

Note: For many of the questions, answers may vary. The answers below are given as guides.

Music Inc. and RadioNow are two companies that carry a new style of mp3 player. During each company's annual executive meeting, they re-evaluated their spending as well as their market prices and profits. Currently, Music Inc. expenses can be modeled by the function $E(x)=500+20x$ where x is the numbers of mp3 players produced. Their profit from each mp3 player is modeled by the function $P(x)=45x$ where x is the number of mp3 players sold at $45 each. The expenses of RadioNow can be modeled by the function $C(x)=500+15x$ where x is the number of mp3 players sold. The profit from each mp3 player made by RadioNow is modeled by the function $M(x)=50x$ where x is the number of mp3 players sold at $50 each.

I. Graph the four functions *E(x), P(x), C(x) and M(x)* on your graphing calculator. Use the following window:

```
WINDOW
 Xmin=-10
 Xmax=40
 Xscl=1
 Ymin=-50
 Ymax=1000
 Yscl=1
 Xres=1
```

Sketch each equation on the graph below, paying careful attention to the y intercept and the points of intersection between all of the lines.

II. Using your graph and graphing calculator, and solving using inverse relationships, answer each of the following questions.

1. What does it mean to *break even*? Which company reaches this point first and how many mp3 players were sold?
   Breaking even is where the company has not made a profit yet but is no longer "in the hole" due to expenses. From this point forward they will make a profit off of any sales. RadioNow reaches this point first by selling about 15 mp3 players.

2. Where does each company break even? What does this look like on the graph? Write the equation used to solve this and then show your work.

   Music Inc. breaks even at (20, 900). This is the point of intersection between E(x) and P(x).
   $500 + 20x = 45x$

   RadioNow breaks even at (14.29,714.29). This is the point of intersection between C(x) and M(x).
   $500 + 15x = 50x$

3. Over time who makes the most profit? How is this seen on the graph? Equations?
   RadioNow makes the most money. This is seen by the slope of the graph. By charging $50 per mp3 player, they will make more money over time then charging $45.

4. Where do the expense equations intersect? What does this mean and how is this reflected in the equation?
   The expense equations intersect at 500. They both start with the same base expense and it is seen as the y-intercept on the graph.

5. Where do the profit equations intersect? How is this reflected in the equations?
   The profit equations intersect at (0,0). You can set them equal to each other and solve for x to find where they are equal. They have the same y-intercept. They are never the same after they sell even one mp3 player.

6. What is the profit if each company sells 100 mp3 players? 200 mp3 players? You may use your calculator to solve the problem but also show the setup and work for solving by hand.
   100 mp3 players: Music Inc. 4500(ignoring expenses) RadioNow 5000

   200 mp3 players: Music Inc. 9000   RadioNow 1000

7. A third company is also selling the new mp3 player for $60. What would the profit equation look like? How will this company compare to the other two? Profit? How is this seen on the graph?

   $y = 60x$. This equation is steeper on the graph than the other two. Profit will be more due to higher charge (slope).

8. If Music Inc. and RadioNow made $2000, how many mp3 players did they sell? (Ignore expenses. Use only profit equations.)
   You can use the equation y = 2000 and find the intersection between it and the profit equations.
   2000 = 50x   2000 = 45x   2000 = 60x
   40 mp3 players   45 mp3 players   34 mp3 players

9. After selling 10 mp3 players, what is each company's profits ignoring expenses again? Is this before or after breaking even?
   Music Inc. is $450 and Radio Now is $500. This is before breaking even for both equations.

10. When does each company make more than $300? Write an inequality to model this situation.
    50x > 300 and 45x > 300 *After selling more than 6 and 7 mp3 players respectively.*

11. When does each company make less than $750? Write an inequality to model this situation.
    50x < 750 and 45x < 750   *Selling up to 15 and 16 mp3 players respectively.*

12. Why is it important to investigate profit vs. expense and also other trends?
    If I am interested in the having my company make the most money possible, then I will try to lower expenses and raise the price for customers to a point that's low enough that they will still buy the mp3 players. (Supply and demand.) Companies are also competitive and will test out different options to see how the customer responds and if consumers will choose their product over the competition's.

13. As a company what do you want to happen to expenses? Profit?
    I want my expenses to decrease and my profits to go up.

15. Which company would you choose to work for and why? What factors affect a company's start-up costs?
    I would pick RadioNow because they have lower expenses and a higher price for customers. They will make the most money over time.

    The factors that affect start-up costs might be business taxes, location, competition, level of skilled workers, distribution costs, return policies and insurance costs.