Area, Surface Area, & Volume: Selected Problems

1. The length of one side of a square is increased by 2 units and an adjacent side is decreased by 2 units. By how many square units do the areas of the original square and the new rectangle differ?



- 2. A solid 3x3x3 cube weighs 5 pounds and is worth \$216. How many dollars is a 4x4x4 cube of the same material worth?
- 3. The length of each side of a cube is increased by 20%. What is the percent increase in the surface area of the cube?
- 4. A rectangular table is 45 inches wide and 64 inches long. A tablecloth hangs over the table 10 inches at each edge. What is the number of square inches in the area of the tablecloth?
- 5. A 6-inch by 6-inch sheet has congruent squares removed from each corner. The sheet is then folded into a rectangular box with no top. What is the maximum possible number of cubic inches in the volume of the box?
- 6. A rectangle that measures 27 centimeters long and 24 centimeters wide is to be covered with square tiles that are 2 centimeters on each side. How many tiles are needed? If we wanted to cover the rectangle with tiles that are 3 centimeters on each side, how many tiles would be needed? If the rectangle is to be covered with both 2- and 3-centimeter tiles, how many of each type would be needed?

Source: Based on problems from http://www.mathcounts.org and from the collection of Grayson Wheatley, Professor Emeritus, Florida State University.

Solutions

- 1. 4 square units. What if the sides are increased and decreased by 3 units? 5 units? *n* units? Algebra can be used to explain this. If the length of a side of the original square is x, what is the area of this square? What are the length, width, and area of the rectangle with sides increased and decreased as described? How do the areas compare?
- 2. \$512. Draw a square of any size. Then draw a square whose sides are twice as long as those of the first square. What is the area of the first square? of the second square? How do they compare? Draw another square with a side three times the length of the first square. How does its area compare to the first two? Try the same experiment with cubes, constructing cubes with side lengths of 1, 2, and 3 units. How do their volumes compare? Can you explain what happens?
- 3. 44% increase
- 4. 5460 square inches
- 5. 16 cubic inches
- 6. 162 2-cm tiles; 72 3-cm tiles. For mixed tiles, 81 2-cm tiles and 36 3-cm tiles is one possibls solution. Are there others?